Intelligent Control
and Cognitive Systems
brings you...

Just Enough About Statistics

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- “I’ve written an algorithm that does X”
- “I ran it on some data and it does better than the standard method”
- “So it’s better, right?”
- “Can I have a first?”

- “Kim and I designed a new user interface and asked Sandy to try it”
- “Sandy found it easier than the old one”
- “So it’s better, right?”
- “Can we share the best thesis prize?”

Actually, no.

Why not?

- Did Sandy drink a lot of coffee that morning? try harder for friends? work with similar interfaces before?
- Would your algorithm still be better on different data? in different network conditions? with different parameters?

Statistics

- Observations are noisy and uncertain.
- They might not turn out the same way twice for all kinds of reasons.
- Statistics is about making inferences when there is noise and uncertainty.
- Where is uncertainty? Everywhere, except logic and pure mathematics.

- We use a *probability model* to express uncertainty about observations
- Divide what we observe into a systematic part, and a random part:
  - $Y = f + \varepsilon$
- Possible examples
  - $f = 9.47$
  - $f(X) = 3.81 + 2.82X$
  - $\varepsilon \sim \text{Normal}(0, 2)$

Example

- Y = 11.34
  - Systematic part (true value) is 9.46
  - Random part ('error') adds 1.88
  - But we don't know that yet.
- We want to know about the systematic part
- So we run experiments...

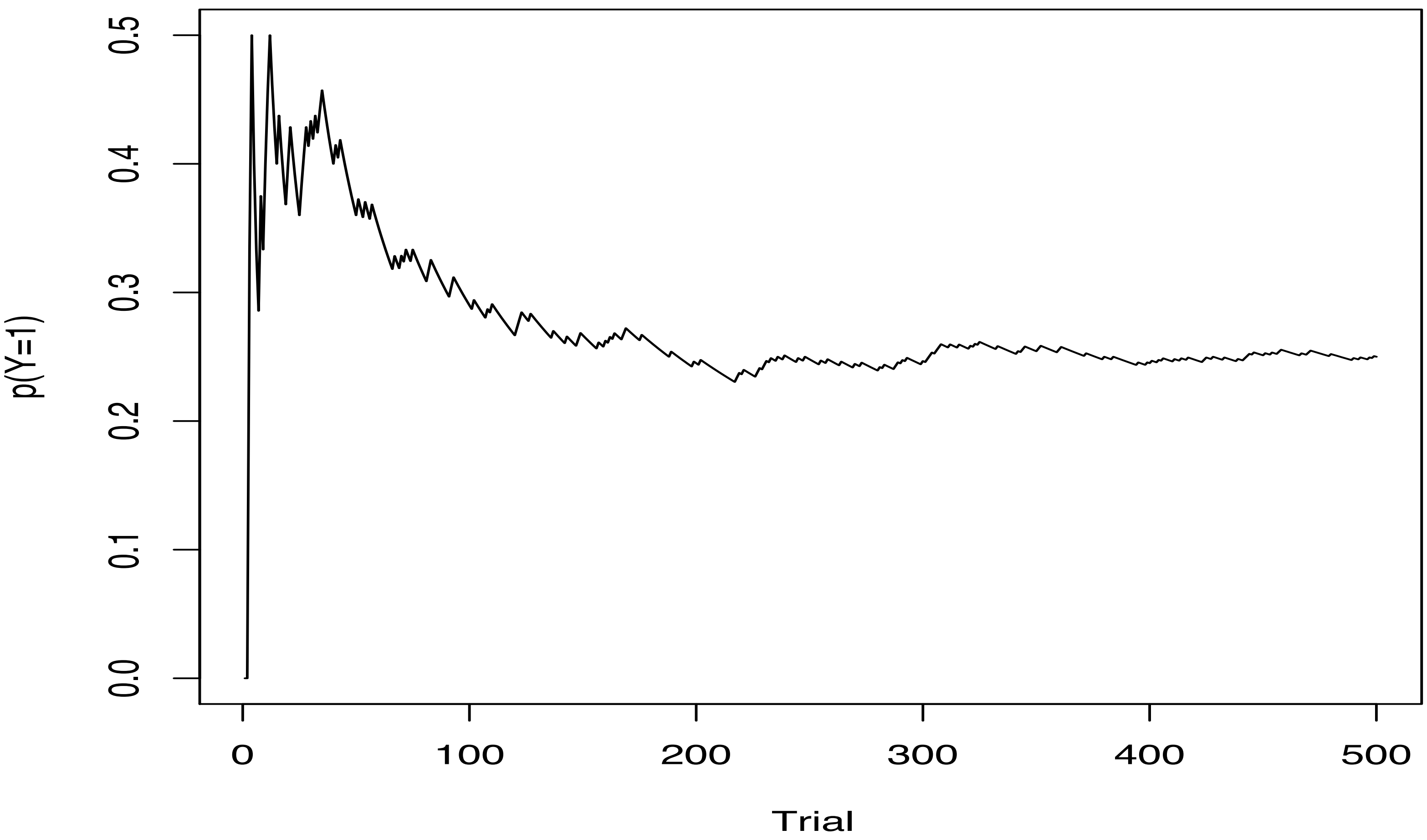
But if it's just random...

- Worry 1: If it's all just random, why take more observations?
- Worry 2: How can we know anything about ε when we don't measure it?
- Answer to Worry 1: The Law of Large Numbers
- Answer to Worry 2: The Central Limit Theorem

Why more is better

- The Law of Large Numbers: "As the number of observations increase, the chance of being very wrong about the systematic part gets very small"
- Example: Suppose in reality:
  - p(success) = p(Y=1) = 0.25
  - p(failure) = p(Y=0) = 0.75
- Let's graph the (frequency of successes) / (number of observations)

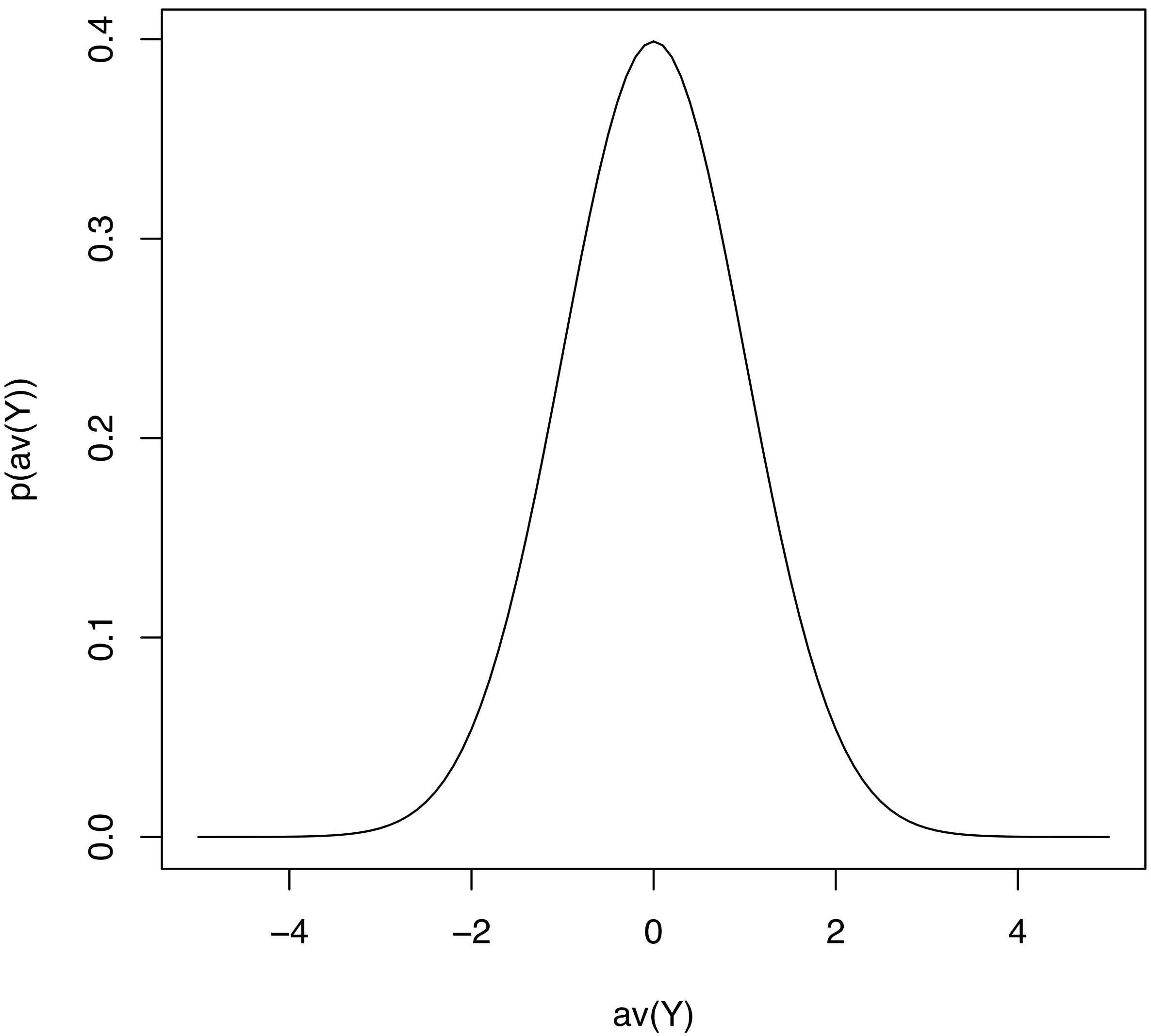
Law of Large Numbers



Noise and Normality

- The Central Limit Theorem (informal): "If ε is the result of many smaller individual 'errors' then the more observations you have, the closer your observed averages are to being Normally distributed"

Noise and Normality



Noise and Normality

- Remarkably, it doesn't matter how the *actual* 'errors' are distributed (effects of coffee, network traffic etc.)
- CLT explains why we often assume normal distributions.

Noise is just signal you haven’t met yet.

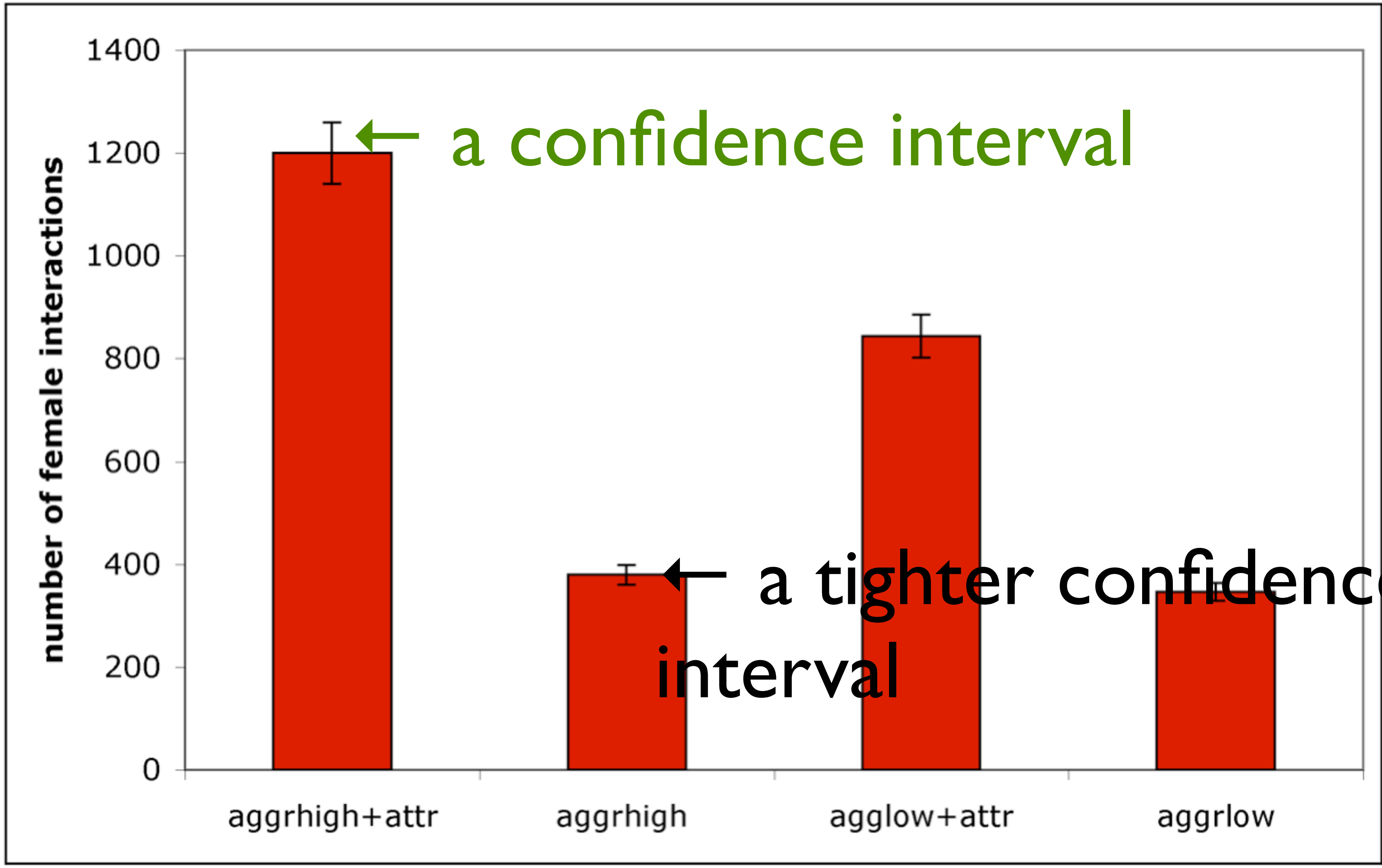
Some applications

- What is the true accuracy of this classifier?
  - Point estimates and confidence intervals
- Is this interface easier to use than the others?
  - Experiments, Hypothesis Tests, and the Analysis of Variance
- What factors affect the performance of this application?
  - Regression

Confidence intervals

- How to estimate what you want to know?
- 'Point estimates' are usually sample averages (cf. law of large numbers).
- In papers you'll hopefully see graphs with confidence intervals, sometimes called 'error bars'.
- They express how confident we can be about the location of the true value.
- Conventionally you'll see 95% intervals.

Number of Fights Involving Females



Confidence intervals

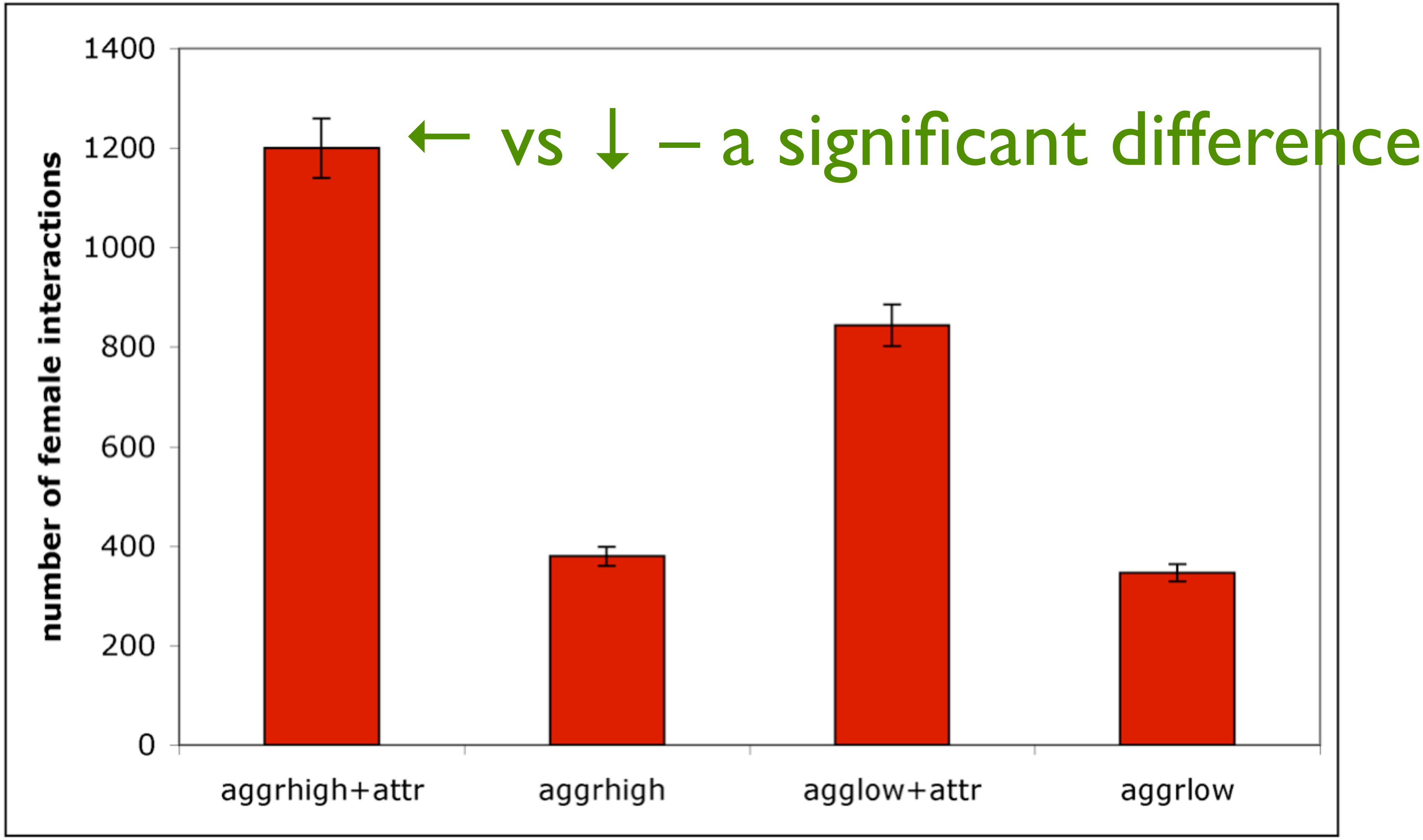
- Confidence intervals come from the variance of the sample average in (theoretical) repeated trials.
- Typically the sample average variance is the sample variance *divided by* the number of observations.
- A 95% interval method comes with a guarantee: If we did this experiment again and again, and computed intervals, then only 5% of them would not contain the true value.
- 99% intervals are wider.

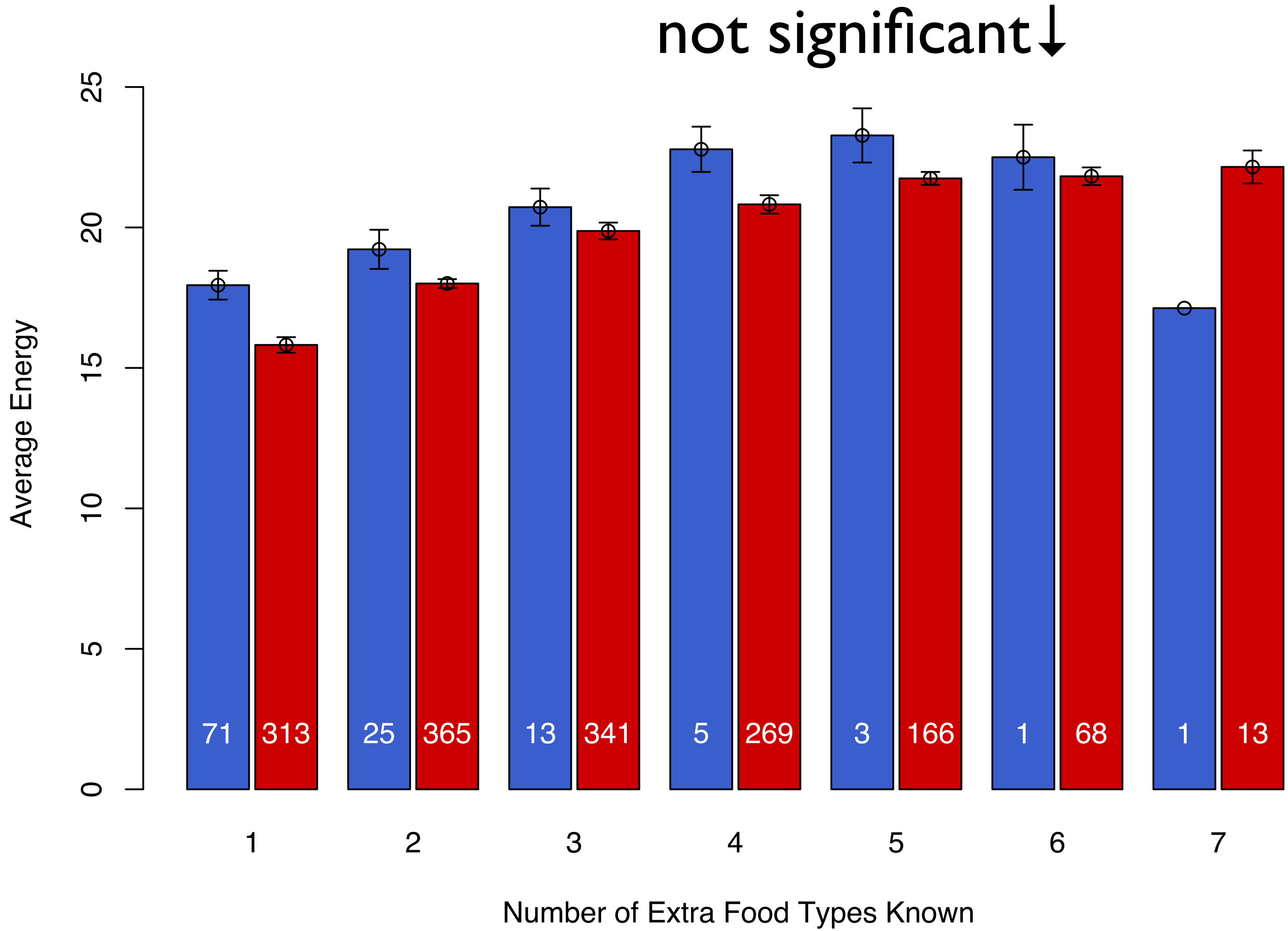
(Why?)

Handy things to do with confidence intervals

- A rule of thumb:
  - If intervals overlap, then the difference in means is not statistically significant
  - If they don't overlap, the difference is statistically significant
- Note: *only* a rule of thumb: check it!

Number of Fights Involving Females





not significant↓
Average Energy
0
5
10
15
20
25
71
313
25
365
13
341
5
269
3
166
1
68
1
13
1
2
3
4
5
6
7
Number of Extra Food Types Known

Confidence & Power

- An unintuitive consequence:
  - How certain you are (how small the standard error || narrow the confidence interval is) does not depend on the size of the population
  - Only the *sample size* and the *variation within the population.*
- It's possible to survey 60,000,00 people with a sample of 1000, e.g. at election time...
- Determining the right sample size requires a power calculation based on effect size & variation.

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Experiments

- Experiments are designed to study causation – what *makes* your program run faster?

  1. Pick subjects, factors, and design.
  2. Randomize and control.
  3. Analyze experimental data in an ANOVA.

     - $Y = \text{mean} + \psi + \varepsilon$
     - ψ is how much Y varies by condition (more on conditions later).

Hypothesis Testing

- ANOVA allows us to *test* for differences by seeing how well our data support:
  - null hypothesis: there is no difference (a=0)
  - alternative: there are differences (a != 0)
- These are only statistical hypotheses, so
  - Cannot say: "these are definitely different".
  - Can say: "these appear significantly different (p<.05)", or "we reject the null hypothesis at the .05 level"

p-values

- When trying to test the hypothesis that a factor makes a difference we can make 2 kinds of mistake:
  - Over-optimism (Type I error),
  - Missed opportunity (Type II error).
- ‘p’ is the probability of thinking you’ve seen a difference when it’s really due to chance (Type I).
- When the p value is small, either there’s a real difference, *or* something unlikely happened.
- p values tell you nothing about Type II errors.

p and statistical significance

- When testing, we pick a measure of statistical significance level or p-value, typically .05
- This is the probability we are willing to risk of making an over-optimistic mistake
- Confidence intervals have the same interpretation: “the true value is within this interval” $p<.05$
- Either the true value is within this interval or something rather unlikely happened...

Statistics means never having to say you're certain...

Experimental Design

- How many subjects do you need?
- Crossed designs and interactions
- Randomization and control

How many subjects?

- Statistical power is 1 - ß (Type II error)
  - ß=The probability of spotting an effect if it's there
  - Higher power experiments are better.
- Normally we fix a Type I error cutoff α (e.g. 0.05), Then power depends on:
  - number of subjects
  - real size of effect (wait, we don't know this!...)
    - importance of pilot studies...

Crossing and interactions

- Good designs get as much information out of as few subjects as possible.
  - e.g. in crossed designs every subject gets every treatment
  - have to randomise over ordering of conditions per subject to check for carry-over effects.
  - Interactions: when the effect of one stimulus is different according to condition (e.g. sex)

Randomisation

- Control: divide subjects into conditions, e.g. sex, year of study; in which you expect effects to *differ*, (should we average effects in men and women?)
  - Within a condition, these factors are fixed.
- Effects can also vary according to *things we don't know about.* Can't control these!
- Randomisation within condition approximately balances subjects with respect to these unobserved factors.

Control what you can,
randomise the rest.

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Regression

- Relationship between performance (Y), network load, time of day and cpu speed.
- Y = b0 + b1 x load + b2 x speed + b3 x time + ε
- Assume we only care about network load. The rest are controls.
    - Get estimates and intervals for b0, b1, b2, b3. Does the interval for b1 overlap 0?
- R^2 measures how much of Y's variance these factors (the model) explain.

Further reading...

- The web is full of free class notes and statistics tutorials e.g.
    - Ian Walker's notes at Bath: http://staff.bath.ac.uk/pssiw/
    - http://davidmlane.com/hyperstat/
- Statistics software packages from BUCS:
    - SPSS, Genstat, Minitab
- But R is best, e.g. https://personality-project.org/r/